CAAM 499 HW 9. DUE BY IN-CLASS WEDNESDAY 11/8

Problem 1 Show that the function $f(x) = \text{sgn}(x)$ is not in $H^1((-1,1)) \subset \mathcal{D}'((-1,1))$. (Take a weak derivative and pair with a test function to see what happens)

Problem 2 Show that the function $f(x) = |x|$, $x \in \mathbb{R}^1$ is in $H^1((-1,1)) \subset \mathcal{D}'((-1,1))$ but it is not in $H^2((-1,1))$. (Hint: Consider $\partial_x^2 T_f$ paired with a text function $\phi \in C_c^\infty((-1,1))$ and split up the integral to see what happens)

Problem 3 Interpret Problem 2 in the context of the Sobolev embedding theorem (which also holds for $H^N((-1,1))$). That is, why does the Sobolev embedding theorem tell you automatically that f is in H^1 but not in H^2?

Problem 4 If $f \in H^k(\mathbb{R}^n)$, show that $\partial_{x_j} f$ is in $H^{k-1}(\mathbb{R}^n)$. Which Sobolev space is $\partial_x^\alpha f$ for $0 \leq |\alpha| \leq k$?