CAAM 499 HW 8. DUE BY IN-CLASS WEDNESDAY 11/8

Textbook exercises in chapter 6:

Exercise 7 (Hint: Do it directly: $\frac{d}{dx}\log|x|(\phi) = -\int \log|x|\phi'(x)dx = -\lim_{\epsilon\to 0}\int_{|x|>\epsilon}\log|x|\phi'(x)dx$. Try integrating by parts. Do the second part similarly.)

Exercise 12 (you don't have to prove your answer. Just state the answer and explain in words why it is true.)

Exercise 24

Exercise 38 (Hint: The problem wants you to show that ϕT is bounded as a distribution. Meaning, for any bounded set B, there is a C and N such that

$$|\langle \phi T, \psi\rangle| \leq C \sum_{\alpha\leq N} ||\partial_x^\alpha \psi||_\infty$$

for any text function ψ supported in B. To make it easier for yourself, you may assume T has order 2.

Problem 1 (not in the text) Say $T \in \mathcal{D}'(\mathbb{R})$ such that T has order ≤ 2, and $\mathrm{supp}(T) \subset \{0\}$. Show that for any test function ϕ that vanishes to order 2 at $x = 0$, we have $T(\phi) = 0$. (Hint: Use the trick in class by defining $b_\epsilon(x) = b(x/\epsilon)$ where $b(x)$ is the bump function equal to 1 on say $[-1, 1]$. The support of T implies $T = b_\epsilon(x)T$ for any ϵ.)